S.J.B.

A. M. D. G.

BULLETIN

of the

American Association of Jesuit Scientists

(Eastern Section)

Published at

LOYOLA COLLEGE

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EASTERN STATES DIVISION

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REV. HENRY W. McLOUGHLIN, S.J.

1872 - 1937

"The souls of the just are in the hands of God, and the torment of death shall not touch them."

In the Roman Missal this text from the Book of Wisdom is quoted in praise of martyrs; yet it is not inappropriate at the beginning of a brief obituary of the late Father Henry W. McLoughlin, S.J., since, at least in a metaphorical sense such expressions as a martyr to science", "a martyr to duty," etc. are quite legitimate. Moreover, the Church recognizes the religious state as an unbloody martyrdom; and in that state, as a priest of a religious Order, he died piously in the Lord. According to Holy Scripture, "Blessed are the dead who die in the Lord." Therefore, he is "blessed". Consequently, he deserves honor and eulogy.

There is, however, something in our gift more valuable to him than eulogy, and it is also in perfect harmony with our consoling dogma of the Communion of Saints. Holy Scripture expresses it thus: "It is a holy and wholesome thought to pray for the dead." Therefore, all friends of Father McLoughlin, who are in the Church Militant, will charitably say the crucifix prayer for the repose of his soul and thus hasten his reward, which is the eternal Beatific Vision of God in the Church Triumphant. Father Harry does not ask us to sculpture the marble to adorn his tomb; he does not desire a monument of bronze to emblazon his Christian virtues, even if such a

memorial could outlast the pyramids. He does, however, desire us to pray for him.

Henry W. McLoughlin was the son of the late Marcus and Catherine Donlon McLoughlin, and was born on the 13th day of September, 1872, in the city of New York. He received his education at Manhattan Academy, conducted by the Brothers of the Christian Schools. Subsequently he entered the College of Saint Francis Xavier. There he successfully completed a course of studies and was graduated from his Alma Mater in June 1892, receiving the degree of Bachelor of Arts. His talents gave promise of a brilliant career in the sphere of the sciences of mathematics and chemistry, and were clearly recognized by his professors. He entered the Society of Jesus in August, 1892, at Frederick, Maryland, where, after two years of Novitiate, he took the three vows of the religious state. During the two subsequent years he reviewed his studies in the classics and rhetoric, and from 1896 to 1899 studied philosophy and science at Woodstock College, Maryland. During his novitiate training he was sent to Washington, D. C., for a month's practice of works of mercy at the Home for the Aged, a charitable institution under the supervision of the Little Sisters of the Poor. An incident occurred there which illustrates Father McLoughlin's truthfulness and genial disposition. One of the pensioners was a gentleman mildly afflicted with senile dementia, whose old blue coat was ornamented with various baubles of brass and colored glass in lieu of distinguished service badges, etc., so dear to the diplomatic corps and to Rajahs and Maharajahs of India. The aforesaid ancient mariner, proudly displaying a fragment of glass, requested Brother McLoughlin to admire the scintillating beauty of his diamond. With the solemnity of Minerva's owl, the novice examined the pseudo-gem, and, with a Mona Lisa smile said: "Mr X., this is a specimen of the mineral called vitrum purum."

It was during Mr. McLoughlin's philosophical course at the Collegium Maximum, Woodstock, Maryland, that his ability in the sciences won for him the high appreciation of the late Father Thomas Freeman, S.J., professor of chemistry, whose advice probably contributed to stimulate the ambition of the young scholastic to specialize in that subject. During his five years of regency, 1899-1904, Mr. McLoughlin taught mathematics and chemistry in Loyola College, Baltimore, and moreover, did strenuous post-graduate study by taking summer courses at Harvard University and at the Massachusetts Institute of Technology. He began his theological course in 1904 at Woodstock, where, on June 28, 1907, he was ordained to the priesthood by Archbishop Farley of New York.

Father McLoughlin was professor of mathematics and chemistry at Georgetown University until 1908, when he was again appointed to Loyola College wherein he taught these same sciences until 1925,

with the exception of an interval of one year, from 1911 to 1912, spent at Saint Andrew on the Hudson, Poughkeepsie, New York, where he devoted himself to ascetical theology and to the study of the Constitutions of the Society of Jesus.

On the feast of the Purification, February 2, 1913, he pronounced his final vows before the altar of the Church of Saint Ignatius, Baltimore, Maryland, and for the next twelve years taught at Loyola College. In 1925 he had given 21 years of his life to the teaching of chemistry and mathematics. Then his health became so seriously impaired that his superiors were constrained to relieve him of the strenuous work in classroom and laboratory. "Never did he return to his beloved teaching; yet teaching's loss became untold gain to parishioners, for during twelve years thereafter he was the devoted parish priest and father confessor to thousands." He was assigned to parish duty at the Church of Saint Ignatius in New York City from 1925 until 1930 where he was conspicuous for his zeal and charity to the poor; and on his return to Baltimore in 1930 continued his apostolic work at our church of the same title.

Here, despite his ever declining strength, he continued valiantly to perform the spiritual and corporal works of mercy until May 19, of the present year when a fatal malady ended his mortal life of praise, reverence and service of Christ the King.

One of his former students, who is now a priest of the Archdiocese of Baltimore, tells us: "Any one who saw Father McLoughlin offering the Holy Sacrifice at the altar, comforting poor weak human nature in the tribunal of penance, visiting the sick, administering the last sacraments to the dying, will testify in glowing terms to the fact that he was a true "*Alter Christus*". All who ever associated with him will accord him the highest praise; for he was the perfect teacher, and the perfect exemplification of Cardinal Newman's definition of a gentleman.

Consider the great number of his former students who are now very prominent in Baltimore and in other cities. They are represented in various walks of life; in business, in the educational sphere, in science, in the legal and medical professions, and in the holy priesthood—and their name is legion."

The alumnus proceeds to narrate what Polonious might call a tragical-comical incident of the famous chemistry class. A Titian-haired student of Celtic lineage attempted an original experiment during the absence of his lynx-eyed professor. Selecting a test-tube he mixed therein three or four samples of chemicals, although he was in Cimmerian ignorance of their latent destructive powers. Then he nonchalantly poured over them some nitric acid. Immediately, fumes like the smoke from Aladdin's lamp issued forth, and a miniature Vesuvius erupted with a direful explosion which shattered the test-tube to smithereens. Uttering winged words, Father Harry rushed

into the laboratory, marched the experimenter to the door and ordered him to read aloud to his classmates a newspaper lyric which he affixed to the adjacent wall. The verses expressed an awful warning to pragmatical omadhauns:

"There was a young chemistry bluff,
Who was mixing a compound of stuff;
He, after a while, threw a match in the vial:
And they found his front teeth and a cuff."

Then the professor gave a quiet little talk to the class, telling them in a fatherly way: "Please give me the name of your coroner, if you persist in being reckless in the laboratory."

On another occasion during a chemistry lecture, the professor noticed that a good student, who had probably been burning the midnight oil, as Erasmus and other intellectual old boys used to do, was nodding in the good old Homeric manner. In fact, he was "Like cowslip wan that hangs the pensive head." Did Father Harry "rage and swell and foam" like to the "ambitious ocean"? Oh, no! He probably remembered Michelangelo's Moses, that glorious marble personification of meekness. So he quietly stationed himself near the tired boy, and sang out like a railroad stentor of ye olden time—"Change C A R S !"

As an illustration of Father McLoughlin's efficiency in teaching, our informant mentions a visit to Loyola by a representative of the State Department of Education. The Dean of Studies invited this gentleman to be present at the class of trigonometry. He accepted the invitation, and was evidently greatly interested and pleased. At the end of the session he expressed with enthusiasm his admiration at the ease and ability of the professor in explaining that branch of mathematics. In fact, he was so well pleased that, when the Dean told him that Father McLoughlin would lecture on chemistry that same afternoon, he preferred to attend it instead of visiting any other class; and so great was his enjoyment therefrom, that it is no exaggeration to say that his amazement and admiration reached its zenith.

The remarkable success that crowned Father Harry's career as moderator of the sanctuary boys, as a teacher at Loyola College, and as a parish priest, is confirmed by the unanimous testimony of many other alumni of Loyola. One, who is now a Major in the Officers' Reserve Corps of the United States Army, declares in his reminiscences of the altar boys and ceremonies in Saint Ignatius' Church: "The Corps of Cadets at West Point are not better trained than we were"—the marching in procession rivalled the best drilled military unit. He considers Father Harry the greatest teacher in technique and ability of all under whom he ever studied in college and university. "He had the faculty of demonstration, the genius of imparting knowledge, the fairness of a Solomon. . . .

As a priest and confessor how shall I attempt to appraise him? I served his Mass for years and learned some of the Latin prayers simply from the precision and decorum with which he said them."

Another alumnus exclaims: "What an excellent moderator he was!" Praising him as a teacher, he too remarks: "I have had many teachers, but not one was so methodical as Father McLoughlin. In his ardent endeavors to impress us with at least a modicum of his assiduous attention to details, he would tell us that he could go in the dark to any part of the laboratory and find any bottle, or apparatus, or paper; and there is not one of us to this day who does not believe that he could readily have done so. He was strict, but he seemed to take a fatherly interest in every one of us. He demanded full attention during his lectures, but they were always well prepared and interesting; and he had withal a keen sense of humor.

If kindness is a virtue, Father McLoughlin was a saint. He was the most kindly of men—a worthy follower of Saint Francis Xavier. And he showed a lively interest in the fortunes—yes, and in the misfortunes of his former boys." Referring to Father Harry's forethought and self-sacrifice in planning, designing, and equipping the laboratories of the new science-building at Loyola, he mentions that, "He was teaching at the time, and also engaged in his priestly duties, but every other working hour was spent on this building which many of the alumni regard as a monument to the memory of one whom the Baltimore diocesan paper has called 'the most beloved man who ever taught at Loyola College.' "

An alumnus who remembers Father McLoughlin especially as his professor of chemistry, portrays him as "a saintly man, an efficient and scholarly teacher."

So great was his reputation in the world of chemistry that, during the investigation of the disaster which occurred when a foreign ship, the 'Alum Chime', loaded with dynamite, blew up in Baltimore harbor, the civil authorities invited Father Harry to give his expert opinion as to the cause of the detonation. For some reason, perhaps from modesty or unwillingness to be shoved into the limelight, he was quoted as replying: 'I am too busy teaching my classes; I cannot spare the time'. His was the happy combination of a teacher dealing meticulously with an exact science, yet relieving the tedium of class by occasional sallies of humor aptly bearing upon the subject, which kept up interest and attention throughout the science lectures. He mentions the little red book containing his problems; how it was kept under lock and key, and as zealously guarded as a tiger's cub. Consequently, when some one told him, perhaps in jest, that access had been gained to its secrets by one of the students, he indignantly denied that such a thing could have happened. Such was his confidence in the integrity of his class. He had a little, sing-song method of indelibly imprinting important

facts on the minds of his students. How many hundreds of times did he sing the refrain: 'All sulphates are soluble save three: lead, barium and stronsium—Please spell that last one correctly for me?'

Other alumni still remember that same refrain after many years devoid of contact with chemistry. "He was a reasonably strict disciplinarian, yet tolerant towards boyish pranks: e.g., blowing air into the gas line and extinguishing all the Bunsen burners, making unearthly smells from weird chemical combinations; and he seemed to be secretly amused at some of their more glaring ineptitudes. In one of his classes was a brilliant mind whose aptitudes lay in other directions than those towards chemistry, who used to be caught nibbling at chunks of deadly chemicals as a quick method of analysis. How Father McLoughlin, with hands upraised in horror, would rush at him; but the youth bore a charmed life and seemed to thrive on the most virulent poisons. The same student occasionally took it into his head to visit the cloister at midnight to bring a chemistry problem to his professor. Father Harry would drive him out like a money-changer from the temple, but with deep chuckles would tell about it the next day."

Many other encomia of Father McLoughlin could be added to this little obituary if space permitted us to quote verbatim the eloquent tributes from the "legion" of his friends who cherish the memory of a priest whom the Eternal Priest has crowned.

In conclusion therefore, one who has presumed to be the necrologist of his good friend and brother in Christ, makes the following unequivocal statement: Father Harry, a man humble, sincere, and of great moral courage, would sternly disapprove of being presented or nominated for a so-called popular canonization such as was conferred, e.g., upon Charlemagne. Nevertheless, inasmuch as Father Harry is forever a priest of the Living God, therefore, in this respect, we call him a greater man than the famous King of the Franks. There was nothing of the "I am Sir Oracle!" about Father Harry: he did not twang a lyre of self-praise; nor did he desire the credit of a great name upon earth such as the world teaches its votaries. No! He honestly strived to conquer the defects of human nature by the supernatural power of God's grace; he tried to conform his life to the "Sum and Scope" of the Constitutions of the Society—For The Greater Glory of God.

Thus he was an obedient religious; he was charitable to the poor; and became worthy to receive the distinguished service cross of "The Kingdom" for his honorable record in the Spiritual combat.

May he rest in peace.

ROBERT F. REYNOLDS, S.J.

REV. ROGER JOSEPH BOSCOVICH, S.J.

1711 - 1787

THE SESQUICENTENNIAL OF A GREAT JESUIT SCIENTIST

Rev. Edward C. Phillips, S.J.,—John G. Furniss, S.J.

In the February, 1937, number of L'Astronomie, the official bulletin of the Societé Astronomique de France (pp. 91-98), George Nikolitch, on the occasion of the 150th anniversary of Father Boscovich's death, calls attention to the remarkable career and works of this "greatest genius Jugoslavia has produced." Believing it would be most proper to recall to our own memory the scientific work of one of whom the Society may be justly proud, we have summarized Nikolitch's article, adding here and there some items from other sources. The opening paragraph reads as follows:

"The Italian Angelo Fabroni has written a very fine life of Boscovich in which, among his other praises of the great Jugoslavian, he calls him 'the commanding genius to whom Rome paid the honor of calling him her master, whom all Italy esteems as a precious treasure, and to whose memory Greece would raise a monument even at the expense of destroying a statue of one of her illustrious heroes.' On reading these eulogies, one might suspect Fabroni of exaggeration, but, once and for all, such a thought must be dismissed as unfair.

"Boscovich has not yet received the place in history that is his due; foreign savants have not judged his accomplishments without bias, and his own countrymen have not sufficiently studied his works to enable them to defend successfully, as other nations do, the priority of their fellow citizen.

"Nevertheless Boscovich is the greatest mind that Jugoslavia has produced, and with his passing there was lost to his country and to the world of Science in general, a universal genius. Many new projects received their start from him; many discoveries were made by him; many inventions came from his hands. Boscovich was mathematician and physicist, astronomer and philosopher, poet and diplomat. He was a member of the Academy of Science of both France and England, and though a Jesuit, was made a noble of the former Italian city-state of Lucca. Towards the end of his life he became a naturalized citizen of France and was appointed Director of the Paris Naval School of Military Optics. He died at Milan on the 13th of February, 1787."

Boscovich was born at Ragusa, (Dubrovnik in Slavic). in Dalmatia on the 18th of May, 1711. At the age of fourteen, he entered the Society of Jesus and was sent to Rome for his studies. He showed such skill in mathematics that he became a professor of that subject even before he finished his theological studies, a post which he filled with great distinction for a whole generation. In accordance with the custom of the time, he published a Latin dissertation each year, evidencing a propensity for astronomy. We find him writing on such topics as "Sunspots" (1736); "The transit of Mercury" (1737); "The Aurora Borealis" (1738); "The applications of the telescope in astronomical studies" (1739); "The figure of the earth" (1739); "The motion of the heavenly bodies in an unresisting medium" (1740); "The Inequality of Gravity in Various Parts of the World" (1741); "The Aberration of the Fixed Stars" (1742).

The Holy Father, Benedict XIV, was quick to make use of his talents. He was consulted about the cracks in the great dome of St. Peter's which were causing widespread consternation, and his plan for surrounding the dome with strong iron bands was successfully adopted. He was the adviser of the Papal Government in all important technical matters, and along with the English Jesuit, Father Christopher Maire, was commissioned to make a survey of the Papal States. This project, of which they published a detailed account entitled "De litteraria expeditione per Pontificiam ditionem ad dimetiendos duos meridiani gradus et corrigendam mappam geographicam" (Rome, 1755) with a new map of the Papal States, not only corrected many previous mistakes, but was of great importance in the discussion of the question, then so prevalent among European scholars, of the figure of the earth.

He also planned and superintended the draining of the Pontine Marshes and the damming of the lakes that threatened Lucca, for which last he was made a noble of that state, and had his expenses paid for his scientific explorations in Italy, France and England.

He first suggested the project of an observatory at the Roman College, which, completed under Father Secchi, later became so well known. The plans for the observatory at Brera, still among the most prominent in Italy, were drawn by him and carried out under his direction.

Many great European Universities sought him as a professor of mathematics, and he was elected to membership in a number of the great scientific societies, among these the Academies of Science of Bologna and Paris, and the Royal Society of London. It was to the latter that he dedicated a general work on Astronomy written in Latin verse entitled: "On eclipses of the sun and the moon" (London. 1760).

The Royal Society. in 1769, proposed to send him in charge of a scientific expedition to California to observe the transit of Venus,

but the storm of opposition then deluging the Society of Jesus caused this proposal to be dropped. It was upon the suppression of the Society in 1773 that he went to Paris, became a naturalized French citizen, and received his appointment to the chair of Optics expressly created for him by Louis XV. He held this for ten years under Louis XV and his successor.

He returned to Milan in 1783 to supervise the printing of his unpublished works. He retired for a while among the Monks of Vallombrosa, but came back to Milan for new work where death followed after severe mental trials.

He also distinguished himself by his political writings, mostly Latin eulogies in honor of the great men of his times, and as a diplomat being called upon to act as his country's ambassador in disputes with both England and France.

Of his many books and treatises, his chief works are: "De litteraria expeditione per Pontificiam ditionem" already mentioned above, "Theoria Philosophiae Naturalis" (1758), and "Opera pertinentia ad opticam et astronomiam maxima ex parte nova et omnia hucusque inedita" (1785).

Boscovich was deeply interested in Physics and Mathematics, as, for example, in the Ovals of Descartes, cycloids, the logistic curve, logarithms of negative numbers, the theory of conic sections, the law of continuity, the limits of certainty in astronomical observations, the solid of greatest attraction, inequalities in terrestrial gravitation, on all of which subjects he wrote scientific treatises.

Though Cotes and De la Caille had given a considerable development to Differential Trigonometry, Boscovich was the first to establish its four essential equations.

He was the predecessor of Lobatchevsky and of Bolyai in the critical discussion of Euclidean geometry, and the first to establish a new geometric system. He was the first one, too, to solve the problem of the solid of maximum attraction. This problem led to research which was intimately connected with the determination of the mean density of the earth and to the outlining of the problem of isostasy. He proposed at least two ways of determining the mean density of the earth by determining the displacement of a plumb line produced by a known quantity of water; for example, the amount of water moved by the ebb and flow of the tide in a narrow inlet, or by the aid of the water that gathers in a valley. He was the first to try, by several actual measurements of a degree of the meridian, to ascertain the polar flattening of the earth. Boscovich was the predecessor, too, of Legendre and Gauss in the development of the fundamental principles of the calculation of errors.

As for the theory of instruments, he was not only the predecessor of Bessel, but he also introduced into this theory a method of solution depending on general differential trigonometry, his treatise

on which subject was accepted for publication in the Memoirs of the French Academy. He also wrote learned treatises on the micrometer and on achromatic telescopes.

It is again Boscovich who, before Bessel, introduced in 1785 the method of coincidences in the pendulum measurements for the determination of the force of gravity.

He was, moreover, the predecessor of Einstein in proposing a general theory of relativity; for he was an integral relativist, being firmly convinced that we can in no way distinguish between absolute and relative motion. He held the same view with regard to position and likewise with regard to quantitative extension, pointing out that if the whole universe were to change its size from day to day whilst keeping the same proportions in all its parts, we would be unable either to measure or even to perceive such expansions and contractions.

Boscovich's interest in Astronomy was first aroused in connection with the stellar aberration which Bradley had recently discovered. By the help of a specially constructed instrument he measured the aberration of the stars β and δ Herculis, and, by a single observation, found for the constant of aberration a value of 20″.*

He offered an elegant solution of the problem of finding the sun's equator and for determining its period of rotation by observation of the spots on its surface.

The astronomers of the eighteenth century were specially interested in comets—and in this field of research the name of Boscovich ranks with that of Lalande, Méchain, and D'Alembert.

Boscovich was the first to propose the principle that in order to determine the orbit of a comet it is necessary to have three observations of position taken at short intervals of time. It was Boscovich also who taught that three observations suffice for determining the orbits of celestial bodies. He was the first, moreover, to announce that the tails of comets are composed of a very fine and light matter which detaches itself from the nucleus of the comet and which forms about it a kind of atmosphere, part of which lengthens out into an extended tail under the action of the sun's heat [radiation].

Basing his studies of Saturn on the work of De Séjour, he explained the reasons for the disappearance of its rings.

We omit here any discussion of his cosmological theories despite their great importance to both philosophy and science, and conclude our very inadequate sketch with an adaptation of the closing paragraph of Nikolitch's article.

"There are many things more we could say about Boscovich concerning his work in mathematics and physics, especially in optics, but it is impossible in a short article to tell all about a man who has

*Note: this probably means "a single set or series of observations." The internationally accepted value is 20″.47.

written some 88 treatises, and who, when he discussed with others his scientific work could refer to its results as already adopted by others and incorporated into the scientific writings of the period. But what we have here told of him is enough to give some idea of his greatness and of his true place in the history of human science. Many historians of science have given to others the credit of priority for truths discovered by him."

The Astronomical Society of Belgrade commemorated in February, 1937, the one hundred and fiftieth anniversary of his death, and is publishing in French and Jugoslavian an account of the life and works of their great compatriot.

It is hoped that this work will soon be at the disposal of those who are interested in the pioneers of Science and in the part the Society of Jesus has taken in such work.

SCIENCE AND PHILOSOPHY

FECHNER'S PSYCHOPHYSICS

REV. W. BURKE-GAFFNEY, S.J.

In the year 1846 Weber announced the law: *The amount the intensity of a physical stimulus must be increased, in order that the increase may be sensed, bears a constant ratio to the intensity of the stimulus.* This means that if A is the intensity of a physical stimulus, and B is the intensity of the next largest that can be sensed to be larger, and C the intensity of the next after B which can be sensed to be larger, then (B-A) A is equal to (C-B) B, which is equal to a constant ratio R.

Weber's Law has been approximately verified, except for very intense and very weak stimuli, in all sensory fields but taste and smell. The constant ratio, of course, is different for the different senses. From Weber's law we obtain the Geometric Series Law, namely: *Within the limits that Weber's Law is true, the intensities of all stimuli that are perceptibly different form a geometric series.* The geometric series law follows inexorably from Weber's Law. For, if (B-A) A equals R, then B equals A x r, where r equals (1+R); and C equals Ar^2; and I_n, the intensity of the nth stimulus perceptibly different from A, equals Ar^{n-1}. It must be noted that the geometric series law does not say that the nth sensation equals Ar^{n-1}, but that the **intensity of the nth stimulus** equals Ar^{n-1}.

The geometric series law enables us to compare the intensity of stimuli if we know their places in the geometric series. For, if

I_n equals Ar^{n-1}, and I_m equals Ar^{m-1}, then $\frac{I_n}{I_m}$ equals r^{n-m}

Pogson, in 1856, made an application of the rule $\frac{I_n}{I_m}$ equals r^{n-m}" to the intensity of the physical stimulus of stars which causes the sensation of brightness, that is, to their luminosity. Hipparchus had graded the stars into six magnitudes. The faintest visible to the naked eye he had called "of the sixth magnitude". The brightest stars in the sky were first magnitude stars. Pogson argued that a fifth magnitude star was the pth perceptibly brighter than a sixth magnitude star; a fourth magnitude one, the 2pth perceptibly brighter; a first magnitude one the 5pth perceptibly brighter than a sixth

magnitude, or, in general, an n^{th} magnitude star was the $(6-n)p^{th}$ perceptibly brighter than a sixth magnitude star.

Hence $\frac{I_n}{I_m}$ equals $r^{(m-n)p}$ where n and m are the magnitudes of the stars, or $\frac{I_n}{I_m}$ equals q^{m-n}, where q equals r^p, which, in turn, equals $\frac{I_1}{I_2}$. The value of q can be determined on the assumption (accepted as fact by astronomers) that the light received from a first magnitude star is 100 times that received from a sixth magnitude, for this gives us q equal to the fifth root of 100, that is equal to 2.512. Pogson's rule then boils down to $\frac{I_n}{I_m}$ equtls 2.512^{m-n}, or $\log\left(\frac{I_n}{I_m}\right)$ equals 0.4 (m-n), where n and m are the magnitudes of the stars whose luminosities I_n and I_m are being compared.

In his *Elements der Psychophysick* (Leipzig, 1860), Fechner expounded the law: *S equals K log I*, where S is the sensation due to the stimulus I, and K is a constant, equal to 1/log r. Fechner's law is generally rejected by psychologists, as not verified by experience. The psychologists might have saved themselves the trouble of attempting to verify Fechner's law if they averted to the fact that, on the very basis of Fechner's deduction, K log I does no equal the sensation.

Fechner's law supposes that we choose our unit for the measurement of intensity of stimulus in such a way that the intensity, I_1, of the first perceptible stimulus will be r, so that I_n will be equal to r^n. Now, if I_n equals r^n, then n equals log I to the base r, but this is equal to log I x 1/log r, that is to K log I. Therefore it is not to the sensation that K log I is equal; and if we give to Fechner's symbols the meaning he gave to them we must reject his law. But we might revise Fechner's law by giving a new meaning to his symbol S which will make the law to have a true meaning, namely: *if the intensity of a stimulus is I, then it is the Sth perceptible, where S is equal to K log I.* To avoid confusion it would, of course, be better to express the law as n equals K log I.

It is worth noting that the rule *n equals K log* I is reduced logically from Weber's Law; and that if Weber's Law is wrong, so is the geometric series law; and that if the geometric series law does not hold for light stimuli, then Pogson's rule is false. But, if Pogson's rule is fallacious, we must say that the foundations of the science of astrophysics are unsound,—a statement that is offensive to pious ears, if mine be pious.

CHEMISTRY

A MICRO VOLUMETRIC DRY COMBUSTION METHOD FOR CARBON

Rev. Richard B. Schmitt, S.J.

Somewhere in the Land of Utopia of the organic chemist, there is an automatic determination for carbon and hydrogen. There are many analysts and research workers roaming around in the Land of Promise searching for this elixir, which is in such demand in the research laboratory.

A simpler micro combustion method than the gravimetric precision method used at present, even though it allows the determination of carbon only, would be a desirable research tool. In the synthesis of new organic compounds a high precision method is not required, and the experimenter merely wants to know whether his

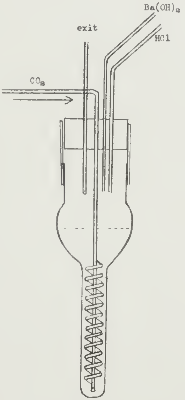

Figure I
Titration Vessel.

work proceeded in the expected direction. A result giving figures within rather liberal limits and of carbon only, would readily indicate whether the expenditure of further labor in form of purification procedures for the final precision analysis is warranted and justified. On the other hand, a result deviating grossly from the expected will indicate to the research worker the fallacy of his work and efforts; indicating too, that there is no need for the purification of the unwanted compound. Then he can proceed immediately to more constructive channels.

Apparatus

As a first step in devising a simple method, it was thought to substitute the gravimetric determination of the carbon dioxide by a volumetric method, using the absorption and titration conditions as devised for small amounts of carbon dioxide by Lindner (1), and as applied by Lieb and Krainick (2) in their wet combustion method. For the actual combustion the apparatus and procedure as described by Niederl and Roth (3) is used. To the capillary outlet of the combustion tube, which is filled according to Pregl, an anhydrone tube is attached. To this anhydrone tube, for the absorption of water, a right angle glass tubing is attached which extends almost to the bottom of the tiration vessel; through this tube the CO_2 flows into the $Ba(OH)_2$ solution. In order to give the CO_2 sufficient time to be absorbed, a glass spiral is attached on the outside of this tube, which fits snugly in the lower portion of the titration flask.

The capillary outlets of the burettes, containing N/10 barium hydroxide solution and N/20 hydrochloric acid, extend through a four-hole rubber stopper into the same titration vessel. An exit glass-tube is attached by rubber tubing to the Mariotte flask, which controls the pressure and flow of gases through the entire train. — The titration vessel is attached to the four-hole rubber stopper by means of soft crepe rubber tubing, so that the $Ba(OH)_2$ solution may be agitated in case of need when the end point is determined.

Chemicals

a) Prepare a N/10 solution of pure $Ba(OH)_2$ according to Lindner (4). In a 2 liter volumetric flask dissolve 32 grams (Kahlbaum) barium hydroxide and 20 grams barium chloride. Shake vigorously; then fill up to the mark, and let stand for 24 hours and filter. Use calcium chloride tube stopper. Determine the factor.

b) Prepare N/20 solution of pure hydrochloric acid. — In a 2 liter volumetric flask place about a liter of distilled water free from CO_2 and allow the contents of one DeHaen Fixanal tube of N/10 hydrochloric acid to flow into the flask and wash down freely with distilled water. Dissolve 60 grams of barium chloride in about 900 cc distilled water, and add this solution to the 2 liter flask containing the acid; mix thoroughly and fill up to the mark.

Let stand for 24 hours and filter. — Determine the factor.
(These solutions are sufficient for more than one hundred determinations.)

c) Determine accurately the factor of the acid and base. (Very Important.)
d) Neutral ethyl alcohol, 95%.
e) Phenolphthalein indicator.

Procedure

1) Set-up combustion train for C & H as modified by Niederl & Roth.
2) Steam titration vessel, glass tube and spiral.
3) Weigh-out sample in platinum boat; not more than 3 mgs., preferably less; otherwise the large amount of precipitate may interfere with the bubbles of CO_2 from moving along the glass spiral.
4) Place boat with sample in combustion tube.
5) Put 0.1 cc neutral ethyl alcohol and 1 drop phenolphthalein in titratic vessel.
6) Connect glass-tube and spiral to inlet tube, i.e. the end of anhydrone tube.
7) The tips of the burettes must be filled with the respective liquids.
8) The inlet-tube with spiral, the burette tips and the inside of the rubber sleeve are thoroughly rinsed with distilled water from micro wash-bottle.
9) Attach the titration vessel to the rubber sleeve which is fastened to the four-hole rubber stopper.
10) Regulate the flow of oxygen through the entire train to 3 cc per minute.
11) Sweep out apparatus with 50 cc of oxygen.
    Take readings of burettes to 0.01 cc.
12) Allow 8 cc of $Ba(OH)_2$ to flow into titration vessel.
13) Begin combustion of sample according to Pregl. — First combustion about twenty minutes; sweeping out about 10 minutes.
14) After combustion and precipitation is complete, increase the pressure and the suction of the Mariotte flask, and sweep out with 100 cc of [illegible]
15) Titrate carefully with HCl. — Read both burettes to 0.01 cc.
16) Calculate per cent.
    Log % C = log $Ba(OH)_2$ + log 0.3 + (1 − log subst).

Sample Calculation

Benzoic Acid

Weight of sample	$Ba(OH)_2$	HCl
3.992	8.08	9.08
1.367	0.13	1.62
2.625 mgs	7.95 cc	7.46 cc

Factor $Ba(OH)_2$	=	1.9904	2.72
Factor HCl	=	1.002	0.38
			2.35 cc
			+7.46
		Total	9.81 cc

7.95 x 1.9904 = 15.8523 cc	log 6.02	77960	
9.81 x 1.002 = 9.83 cc	Log 3	47712	
	antelog wt. sam.	58087	
6.02 cc			
		83759	= 68.80%
		Thoer.	= 68.88%

Results

Substance	Sample mg	% C Found	% C Calc.	Error %
1. Resorcinol	2.799	65.17	65.45	—0.28
2. Resorcinol	2.595	65.25	65.45	—0.20
3. Resorcinol	2.696	65.52	65.45	—0.07
4. Resorcinol	2.540	65.32	65.45	—0.13
5. Benzoic Acid	2.625	68.80	68.88	—0.08
6. Benzoic Acid	3.390	68.88	68.88	0.00
7. Benzoic Acid	2.694	68.73	68.88	—0.15
8. Benzoic Acid	2.535	69.10	68.88	—0.22
9. Diphenyl	2.803	93.31	93.50	—0.19
10. Diphenyl	3.445	93.24	93.50	—0.26

Conclusion

This method for determination of carbon in organic substances is independent of climatic conditions, since it is done in a closed system; weighing of absorption tubes is eliminated; the time factor is the same as the Pregl combustion method.

In further investigations we hope to eliminate the colorimetric determination of the end point, and substitute a conductivity method using the potentiometer.

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MATHEMATICS

SOME APPLICATIONS OF COMPLEX QUANTITIES TO CIRCULAR (TRIGONOMETRIC) QUANTITIES

Rev. Thomas J. Love, S.J.
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In verifying trigonometric identities involving terms of the type $(\sin py)^m (\cos qy)^n$, where m and n are positive integers and p and q are any real numbers, the use of the exponential form of the sine and cosine, sc.

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}; \text{ and } \cos x = \frac{e^{ix} + e^{-ix}}{2}$$

is in general simpler, and by actual test, offers a time-saving feature which should commend it to our use.

The advantage of using the exponential form of the sine and cosine is even greater in evaluating the definite integral of a trigonometric function involving terms of the above mentioned type.

Let us define, as is usual in treatises on Analysis (I)

(1) $\sin x = x - x^3/3! + x^5/5! - x^7/7! + \ldots\ldots$

(2) $\cos x = 1 - x^2/2! + x^4/4! - x^6/6! + \ldots\ldots$

where x is real.

Further it can be shown that (II)

(3) $e^z = 1 + z + z^2/2! + z^3/3! + \ldots\ldots$

where z is a complex quantity. If z is pure imaginary, then

(4) $e^z = e^{ix}$,

where x is real and $i = \sqrt{-1}$.

Then from (3) we have

(5) $e^{ix} = 1 + ix - x^2/2! - ix^3/3! + x^4/4! + \ldots\ldots$

Multiplying (1) by i

(6) $i \sin x = ix - ix^3/3! + ix^5/5! - ix^7/7! + \ldots\ldots$

Adding together equations (2) and (6), we have,

(7) $\cos x + i \sin x = 1 + ix - x^2/2! - ix^3/3! + x^4/4! + \ldots\ldots$

Comparing equations (5) and (7) we have

(8) $\cos x + i \sin x = e^{ix}$.

By a similar procedure it can be shown that

(9) $\cos x - i \sin x = e^{-ix}$.

Subtracting (9) from (8):

$$2i \sin x = e^{ix} - e^{-ix};$$

$$(10) \quad \therefore \sin x = \frac{e^{ix} - e^{-ix}}{2i};$$

similarly,

$$(11) \quad \cos x = \frac{e^{ix} + e^{-ix}}{2}.$$

It can be shown (III)

$$(12) \quad d\,dx\,(e^{(a + ib)x}) = (a + ib)\,e^{(a + ib)x} \text{ and}$$

$$(13) \quad \int e^{(a + ib)x}\,dx = \frac{e^{(a + ib)x}}{a + ib}$$

where a, b, are real constants and x a real variable. If a is zero, then equation (12) becomes

$$(14) \quad d\,dx\,(e^{ibx}) = ibe^{ibx} \text{ and equation (13) becomes}$$

$$(15) \quad \int e^{ibx}\,dx = e^{ibx}\,ib.$$

Using equation (8): $e^{ix} = \cos x + i \sin x$, we obtain,

$$(16) \quad e^{2\Pi ni} = \cos 2\Pi n + i \sin 2\Pi n = 1.$$

$$(17) \quad e^{(2n + 1)\Pi i} = \cos (2n + 1)\Pi + i \sin (2n + 1)\Pi = -1$$

Using equation (9): $e^{-ix} = \cos x - i \sin x$, we obtain,

$$(18) \quad e^{-2\Pi ni} = \cos 2\Pi n - i \sin 2\Pi n = 1.$$

$$(19) \quad e^{-(2n + 1)\Pi i} = \cos (2n + 1)\Pi - i \sin (2n + 1)\Pi = -1.$$

The following examples, done in detail, illustrate the use of the more important of the foregoing equations, viz.,

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\int e^{ibx}\,dx = \frac{e^{ibx}}{ib}$$

Verification of the trigonometric identity (IV)

$$(20) \quad \sin x(8 \cos^3 x - 4 \cos x) = 4x$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}; \quad \cos x = \frac{e^{ix} + e^{-ix}}{2}$$

Substituting these values, the lefthand side of Equation (20) becomes

$$\frac{e^{ix} - e^{-ix}}{2i} \left\{ 8 \left[\frac{e^{ix} + e^{-ix}}{2} \right]^3 - 4 \left[\frac{e^{ix} + e^{-ix}}{2} \right] \right\} =$$

$$\frac{e^{ix} - e^{-ix}}{2i} \left\{ e^{3ix} + 3e^{2ix}e^{-ix} + 3e^{-2ix}e^{ix} + e^{-3ix} - 2e^{ix} - 2e^{-ix} \right\} =$$

$$\frac{e^{ix} - e^{-ix}}{2i} \left\{ e^{3ix} + e^{ix} + e^{-ix} + e^{-3ix} \right\} =$$

$$\frac{1}{2i} \left\{ e^{4ix} - e^{2ix} + e^{2ix} - e^{0} + e^{0} - e^{-2ix} + e^{-2ix} - e^{-4ix} \right\} =$$

$$\frac{1}{2i}\left\{e^{4ix} - e^{-4ix}\right\} = \sin 4\,x$$

Hence

Sin x (8 cos³x − 4 cos x) = sin 4 x.

Verify (V)

(21) $\frac{1}{2}\sin(n+m)\,x - \frac{1}{2}\sin(n-m)\,x = \cos nx \sin mx$. Substituting the exponential form for the sine and cosine, the left hand side of equation (21) becomes

$$\frac{1}{2}\left\{\frac{e^{i(n+m)x} - e^{-i(n+m)x}}{2i}\right\} - \frac{1}{2}\left\{\frac{e^{i(n-m)x} - e^{-i(n-m)x}}{2i}\right\} =$$

$$\frac{1}{4i}\left\{e^{inx}.\,e^{imx} - e^{-inx}.\,e^{-imx} - e^{inx}.\,e^{-imx} + e^{-inx}.\,e^{imx}\right\} =$$

$$\frac{1}{4i}\left\{e^{inx}\,(e^{imx} - e^{-imx}) + e^{-inx}\,(e^{imx} - e^{-imx})\right\} =$$

$$\frac{1}{4i}\left\{e^{imx} - e^{-imx}\right\}\left\{e^{inx} + e^{-inx}\right\} =$$

$$\frac{1}{2i}\left\{e^{imx} - e^{-imx}\right\}.\ \frac{1}{2}\left\{e^{inx} + e^{-inx}\right\} =$$

sin mx cos nx

$\therefore\ \frac{1}{2}\sin(n+m)\,x - \frac{1}{2}\sin(n-m)\,x = \cos nx \sin mx$

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(22) $\displaystyle\int_0^{\pi/2} \sin^6 2\,\theta \quad d\theta$ (VI)

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

Here $x = 2\,\theta$

$$\therefore\ \sin 2\,\theta = \frac{e^{2i\theta} - e^{-2i\theta}}{2i}$$

$$\sin^6 2\,\theta = \frac{1}{64i^6}\left\{e^{2i\theta} - e^{-2i\theta}\right\}^6 = -\frac{1}{64}\left\{e^{2i\theta} - e^{-2i\theta}\right\}^6.$$

Expanding according to the binominal theorem:

$$\sin^6 2\,\theta = -\frac{1}{64}\left\{e^{12i\theta} + 12e^{10i\theta}\,e^{-2i\theta} + 15e^{8i\theta}\,e^{-4i\theta} - 20e^{6i\theta}\,e^{-6i\theta}\right.$$
$$\left. + 15e^{4i\theta}\,e^{-8i\theta} - 12e^{2i\theta}\,e^{-10i\theta} + e^{-12i\theta}\right\}.$$

Simplify the above expression.

Then, integrating term by term and remembering that $-\frac{1}{64}$ multiplies each term, we have

$$\int_0^{\pi/2} e^{12i\theta}\,d\theta = \left[\frac{1}{12i}\,e^{12i\theta}\right]_0^{\pi/2} = \left[\frac{1}{12i}\left\{1 - 1\right\}\right] = 0$$

$$12\int_0^{\Pi/2} e^{8i\theta}\, d\theta = \left[\frac{12}{8i}\, e^{8i\theta}\right]_0^{\Pi/2} = \frac{3}{2i}\left\{1 - 1\right\} = 0$$

$$15\int_0^{\Pi/2} e^{4i\theta}\, d\theta = \left[\frac{15}{8i}\, e^{4i\theta}\right]_0^{\Pi/2} = \frac{15}{4i}\left\{1 - 1\right\} = 0$$

$$-20\int_0^{\Pi/2} d\theta = \left[-20\,\theta\right]_0^{\Pi/2} = -10\,\Pi$$

$$15\int_0^{\Pi/2} e^{-4i\theta}\, d\theta = \left[\frac{15}{-4i}\, e^{-4i\theta}\right]_0^{\Pi/2} = -\frac{15}{4i}\left\{1 - 1\right\} = 0$$

$$\therefore \int_0^{\Pi/2} \sin^6 2\,\theta\, d\theta = \frac{5\Pi}{32}$$

DeMoire's Theorem (VII)

(23) $(\cos x \pm i \sin x)^n = \cos nx \pm i \sin nx$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \qquad \cos x = \frac{e^{ix} + e^{-ix}}{2}$$

From equation (8)

$$\frac{e^{ix} + e^{-ix}}{2} + \frac{i\,(e^{ix} - e^{-ix})}{2i} = e^{ix}$$

From equation (9)

$$\frac{e^{ix} + e^{-ix}}{2} - \frac{i\,(e^{ix} - e^{-ix})}{2i} = e^{-ix}$$

$\therefore$ $(\cos x \pm i \sin x)^n = (e^{\pm ix})^n = e^{\pm inx}$

Hence the lefthand side of equation (23) $= e^{\pm inx}$

Also $\cos nx + i \sin nx = \dfrac{e^{inx} + e^{-inx}}{2} + \dfrac{i\,(e^{inx} - e^{-inx})}{2i} = e^{inx}$

and

$$(\cos nx - i \sin nx) = \frac{e^{inx} + e^{-inx}}{2} - \frac{i\,(e^{inx} - e^{-inx})}{2i} = e^{-inx}$$

$\therefore$ $\cos nx \pm i \sin nx = e^{\pm inx}$.

Hence the righthand side of equation (23) $= e^{\pm inx}$

Hence $(\cos x \pm i \sin nx)^n = \cos nx \pm i \sin nx$.

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THE PROBLEM OF THE DOG AND THE RABBIT

REV. FREDERICK W. SOHON, S.J.

The following problem comes up for solution every so often, and it may be worth while having a solution on hand. One familiar with ordinary differential equations should have no difficulty with the solution; but it was thought that the following solution might afford the readers of THE BULLETIN a useful exercise in hyperbolic functions. The reader should consult formulas 652-704 in Pierce, "A Short Table of Integrals."

The Problem: When the dog first sees the rabbit, the latter is due North of the dog and runs due East with a constant velocity. The dog runs with constant speed, but always toward the rabbit. Let it be supposed that the speed of the rabbit is u, and the speed of the dog is to the speed of the rabbit as m: 1, and that the initial distance between dog and rabbit is given as a. It is required to determine the path of the dog, and how long it takes to catch the rabbit.

The solution: Take the origin at the position from which the dog first saw the rabbit. Then at the instant t, the position of the rabbit will be:

$$x = ut \qquad y = a \tag{1}$$

The dog starts at the origin, and runs with constant speed

$$\frac{ds}{dt} = mu \tag{2}$$

and always runs directly toward the rabbit, so that

$$\frac{dy}{dx} = \frac{a - y}{ut - x} \tag{3}$$

The solution will be carried out with hyperbolic functions. If θ is the azimuth of the dog's velocity at any instant and we introduce (Pierce, 704, 702, 701)

$$\begin{cases} z = \log \tan (45^\circ + 1/2\, \theta) \\ \theta = \text{gd}\, z \end{cases} \tag{4}$$

Then

$$\begin{cases} \dfrac{dx}{ds} = x' = \sin \theta = \tanh z & (5) \\[2ex] \dfrac{dy}{ds} = y' = \cos \theta = \text{sech}\, z & (6) \end{cases}$$

and dividing (5) by (6) member by member

$$\frac{dx}{dy} = \frac{x'}{y'} = \sinh z \tag{7}$$

so that from (7) and (3) $\quad ut - x = (a - y) \sinh z. \qquad (8)$

Note that we have from (2) and (7)

$$\frac{dt}{ds} = 1 \; mu \quad \text{and} \quad x' = y' \sinh z \qquad (9) \;\&\; (10)$$

Then differentiating (8) with respect to s

$$1 \; m - x' = (a - y) \cosh z \;\; z' - y' \sinh z \qquad (11)$$

and with the aid of (9) and (10) we find

$$m \, (a - y) = \frac{\operatorname{sech} z}{z'} \qquad (12)$$

Differentiating (12) with respect to s, and using (6)

$$- my' = - M \operatorname{sech} z = - \tanh z \operatorname{sech} z - \operatorname{sech} z \;\; z'' \; z'^2 \qquad (13)$$

the factor z′ cancelling from numerator and denominator in the first term of the right hand member of (13).

$$\text{Then} \quad \frac{z''}{z'} = mz' - \tanh z \;\; z' \qquad (14)$$

and integrating we obtain

$$\log z' = m \, z - \log \cosh z - \log C$$

$$\text{which may be written} \quad z' = e^{mz} \; \operatorname{sech} z \, C = \frac{dz}{ds} \qquad (15)$$

$$\text{whence } ds = e^{-mz} \cosh z \, dz = mu \, dt \qquad (16)$$

$$dx = \tanh z \, ds = C \, e^{-mz} \sinh z \, dz \qquad (17)$$

$$dy = \operatorname{sech} z \, ds = C \, e^{-mz} \, dz \qquad (18)$$

with the aid of (9), (5) and (6).

Integrating by parts, we find

$$\int e^{-mz} \sinh z dz = e^{-mz} \cosh z + m \int e^{-mz} \cosh z \, dz$$

$$\int e^{-mz} \cosh z \, dz = e^{-mz} \sinh z + m \int e^{-mz} \sinh z \, dz$$

ane solving for the desired integrals, we obtain

$$\int e^{-mz} \sinh z \, dz = - e^{-mz} (\cosh z + m \sinh z) \quad (m^2 - 1) \qquad (19)$$

$$\int e^{-mz} \cosh z \, dz = - e^{-mz} (m \cosh z + \sinh z) \quad (m^2 - 1) \qquad (20)$$

We can obtain a more compact form by substituting $m = \coth w$ (21)

whence $\cosh w = m \quad \sqrt{m^2 - 1}, \ \sinh w = 1 \quad \sqrt{m^2 - 1}$ (22)

then (19) and (20) become

$$\int e^{-mz} \sinh z \, dz = - e^{-mz} \sinh w \sinh (z + w) \qquad (23)$$

$$\int e^{-mz} \cosh z \, dz = - e^{-mz} \sinh w \cosh (z + w) \qquad (24)$$

so that the integrals of (16), (17) and (18) are

$$s = C_1 - C \, e^{-mz} \sinh w \cosh (z + w) \qquad (25)$$

$$x = C_2 - C \, e^{-mz} \sinh w \sinh (z + w) \qquad (26)$$

$$y = C_3 - C \, e^{-mz} \tanh w \qquad (27)$$

Taking the equations

$$z' = e^{mz} \operatorname{sech} z \quad C \qquad (15 \text{ bis})$$

$$y = C_3 - C \, e^{-mz} \quad m$$

the latter being obtained from (27) by means of (21)

with $m(a - y) = \operatorname{sech} z \quad z'$ (12 bis)

eliminate y and z' obtaining $m(a - C_3) + C e^{mz} - C e^{-mz}$
showing that $C_3 = a$. (29).

The initial conditions are:

$$s = t = x = y = \theta = z = 0 \qquad (30)$$

whence at the start $\sin\theta = \tanh z = 0$

$$\cos\theta = \operatorname{sech} z = 1 \qquad (31)$$

and putting (30) and (31) in (25) (26) and (27) we find

$$\begin{cases} 0 = C_1 - C \sinh w \cosh w \\ 0 = C_2 - C \sinh^2 w \\ 0 = a - C \tanh w \end{cases}$$

so that $C = a \coth w = a m$ (32)

$C \sinh w = a \cosh w$ (33)

$C_1 = a \cosh^2 w$ (34)

$C_2 = a \sinh w \cosh w$ (35)

and from (25), (26), (27) and (2)

$$s = a \cosh w \left[\cosh w - e^{-mz} \cosh (z + w)\right] \qquad (36)$$

$$x = a \cosh w \left[\sinh w - e^{-mz} \sinh (z + w)\right] \qquad (35)$$

$$y = a (1 - e^{-mz}) \qquad (38)$$

$$t = s / mu = \frac{a \sinh w}{u}\left[\cosh w - e^{-mz} \cosh (z + w)\right] \qquad (39)$$

When the dog seizes the rabbit, $y = a$, and z becomes infinite.

However $e^{-mz} \sinh z = 1/2 \left[e^{-(m-1)z} - e^{-(m+1)z}\right]$
and as both exponents are negative for m greater than unity, both terms vanish. Similarly the term containing cosh z will vanish.

At the instant of seizure, then

$$s = a \cosh^2 w = am^2/(m^2 - 1) \qquad (40)$$

$$x = a \sinh w \cosh w = am/(m^2 - 1) \qquad (41)$$

$$y = a$$

$$t = a \sinh w \cosh w/u = am/u(m^2 - 1) \qquad (42)$$

To obtain an equation involving x and y only, we obtain from (38)

$$e^{-mz} = 1 - \frac{y}{a}$$

$$e^{-z} = \left(1 - \frac{y}{a}\right)^{\frac{1}{m}}$$

$$\frac{m}{m^2 - 1} - \frac{x}{a} = e^{-mz}\left[m \cosh z + m^2 \sinh z\right] / (m^2 - 1)$$

then this becomes:

$$1 - \frac{x}{\frac{am}{m^2 - 1}} = 1/2\,(m + 1)\left(1 - \frac{y}{a}\right)^{\frac{m-1}{m}} - 1/2\,(m - 1)\left(1 - \frac{y}{a}\right)^{\frac{m+1}{m}}$$

If we take the point of seizure as the origin and let

$$\begin{cases} X = \dfrac{a\,m}{m^2 - 1} - x \\ Y = a - y \end{cases}$$

then the equation of the path can be written

$$\frac{X}{a\,m} = \frac{Y}{2}\left[\frac{1}{m-1}\sqrt[m]{\frac{a}{Y}} - \frac{1}{m+1}\sqrt[m]{\frac{Y}{a}}\right]$$

It will be noted that the distances, and the path are all independent of the speed of the rabbit u.

PHYSICS

THE NEUTRINO.

An historical summary indicating the reasons which led to its postulation and a consideration of the indirect experimental evidence for its objective reality.

REV. JOHN S. O'CONOR, S.J.

I. Introduction

In the years immediately following the discovery of radioactivity by Becquerel (1) the radiations from radio-active bodies attracted attention primarily because of their marked effect upon a photographic emulsion; it was in fact this very property, which resulted in the fortunate accident of a fogged plate, that opened up a new realm of scientific investigation which today is the basis of Nuclear Physics.

As early as 1899 Rutherford (2) differentiated between alpha and beta radiation, using as a criterion the penetrating powers of the two types of rays; and in 1900 Villard (3) reported the existence of the third type and the most penetrating of the three, which he named gamma rays.

In 1909 Bucherer (4) confirmed what had been previously surmised; namely that the so-called beta radiation consisted of electrons which were projected at high speeds from the radioactive matter.

However up to the year 1910 most of the investigation concerned with beta rays was confined to the measurement of the absorption and reflection of these particles in and from various materials.

The absorption method of studying beta radiation consists essentially in the measurement of the ionization due to the beta rays, after they have passed through successively increasing thicknesses of some absorbing material such as Aluminum or Mica. By this method the maximum velocity (and energy) of the rays from any body can be estimated from the effective range in the absorber; for beyond a certain thickness the electrometer current, which measures the ionization, drops off suddenly, and the corresponding kink in the curve of intensity vs. thickness is taken as an indication as the effective range of the electrons in question. From the range-energy relation established by such workers as Schonland (5) Varder (6) and Madgwick (7), who used homogeneous groups of rays of known energy to establish this relation, the energy of electrons of specific penetrating power may be deduced.

Although Sargent (8) and Chalmers (9) have developed methods for determining inhomogeneous distributions from absorption curves, the methods are very laborious and indirect and do not yield a high degree of accuracy.

In 1907 Schmidt (10) showed by both absorption and magnetic deflection methods (to be discussed later) that the beta rays from Radium E were given off not as a monokinetic beam but rather in a way that indicated the existence of a continuous velocity spectrum, with an upper limit as high as 95% of the velocity of light.

There was considerable difference of opinion concerning the homogeniety of the beta rays from most other radioactive substances at this time, and to check the work done by absorption methods von Beayer, Hahn and Meitner (11) performed a series of illuminating experiments in which a magnetic field was applied at right angles to a beam of beta rays which had been partially collimated by a slit system. The field bent the beam into the arc of a circle whose radius was determined by the speed of the particles and the strength of the magnetic field, and the particles thus deflected were detected by means of a photographic plate placed in their path.

When using a source of Mesothorium 2 the above workers found that instead of a single slit image (which would have been the result if all the particles had travelled to the plate with the same velocity) there were several well defined lines, indicating not the velocity smear suggested by Schmidt for Radium E, but rather individual homogeneous groups of particles with their own distinct and proper velocities, constituting what has since been characterized as the beta ray line spectrum of the radioactive body in question.

The discovery of these homogenous groups led to a further study of beta ray emission by magnetic deflection methods. Danysz (12) was the first to introduce a modification of the simple method of direct deviation. His new instrument was found to be a vast improvement, especially in regard to the focusing action produced, and the principle was used in the somewhat later development of Rutherford and Robinson (13). This instrument was named by Wooster (14) the Magnetic spectrometer.

The spectrometer consisted of a rectangular box capable of being evacuated, and built to fit between the poles of a magnet. As can be seen from the diagram (Fig. 1) the radioactive source is so situated in this box that the emitted charged particles (under proper conditions of polarity) will describe semi-circles (if the magnetic field be homogeneous) and if a photographic plate be placed in the same plane as the slit which defines the beam of particles, then the particles which emerge from the slit with identical velocities are concentrated along a well defined edge on the plate. This edge constitutes the line from which measurements may be made, and from which

the radius of curvature of the particles in question may be readily deduced.

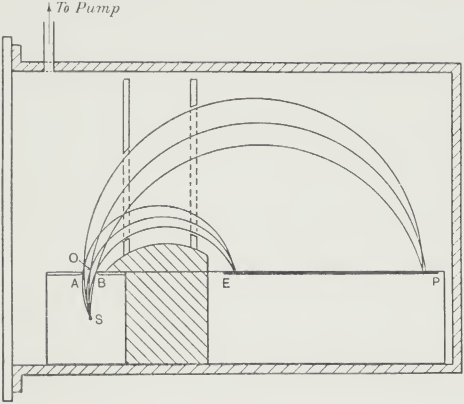

Figure I
Magnetic Spectrometer.

Knowing the radius of curvature ρ in a given magnetic field the momentum and energy of the particle may then be calculated from the relations:

$$H\rho = \frac{c}{e}\,mv = \frac{1}{e}\left(E^2 + 2mc^2E\right)^{1/2} \qquad (1)$$

where H = magnetic field strength in Oersteds; c = the velocity of light; e, m and v = the charge, mass and velocity respectively of the electron, and E = its kinetic energy.

Wooster (14) has elaborated the theory of the focusing action in detail and has shown that there is considerable lack of symmetry in the lines, but that the maximum for any homogeneous group occurs at a distance behind the far edge, or high velocity limit, equal to the diameter of the source.

Hartree (15) has investigated the effect of inhomogenities in the magnetic field on the path of the particles and has shown that such variations produce their maximum effect half way along the trajectory, and are a minimum at the positions of source and detector. He also gives a method for determining the value of the homogeneous field which would produce the same effect as the inhomogeneous one

under consideration, thus affording a means of determining the momentum of the particles despite these irregularities.

This instrument when used with the photographic plate as described above is most useful for the study of line spectra, and with its aid Rutherford Robinson and Rawlinson (16) showed that similar homogeneous groups of beta rays could be produced by the action of gamma rays on sheaths of metal wrapped around a radium B + C source, the walls of whose container were sufficiently thick to stop practically all of the line spectrum beta radiation coming from the source proper.

This result showed by analogy that the electrons forming what was then considered the primary, or line spectrum of radioactive elements, was in reality the result of a secondary process.

In the light of later knowledge, which followed from the above discovery, we may now summarize what is known of the process by which the line spectrum is produced.

II

Theory Of Beta Ray Line Spectra

From the constancy of nuclear masses (and hence total energy) as measured by the mass spectograph as well as from alternating intensities in band spectra we have good evidence that the constituents of the normal radioactive atom exist in quantized states. The general homogeniety of the alpha ray energies, and the existence of discreet energy groups known as fine structure where multiple alpha ranges do appear, indicate that the same must be true for nuclei in excited states.

From the work of Meitner (17) we also know that the primary event in any radioactive disintegration is the emission of a high speed particle,—in the case under consideration an electron. Whether such an electron actually existed in the nucleus prior to its emission will be considered later. At present our interest is in the fact that its expulsion frequently leaves the nucleus in an excited state and that in settling down into the normal state the emission of gamma rays takes place. Many of these gamma rays escape entirely from the atom and are recognized experimentally by their high penetrating power and immunity from deflection in powerful magnetic fields. A fraction of them are however absorbed photoelectrically by the electrons outside the nucleus which constitute the K. L. M. etc. shells of the atom which emits them.

This absorption of the gamma rays results in the further expulsion of an electron from one of the above mentioned shells, the energy of this expelled electron being a function both of the frequency of the gamma ray and of the energy level from which the electron is extracted. Thus a nucleus emitting a gamma ray of one frequency may project secondary electrons with as many different speeds as there are stationary states in the extra-nuclear electronic systems,

and gamma rays of n different frequencies may project electrons with n times as many different speeds.

This process is called internal conversion and is quite distinct from the initial particle emission preceding the gamma ray which gives rise to what are known as the true disintegration electrons. The energy of the electron which is the result of internal conversion is given by the difference between the energy of the exciting gamma ray and the energy necessary to extract the electron from the particular shell in which it was residing. Since as the very name implies the line spectrum energy is quantized we can find the wave length of the exciting gamma ray from the quantum relation $E = h\nu - K$; $h\nu - L_{I}$; $h\nu - L_{II}$, where E is (as before) the electron energy, h is Planck's constant, and K, L_{I}, L_{II}, etc. represent the work required to extract the electron from the respective K or L shells and ν is the frequency of the gamma radiation. The complete theory of internal conversion may be found in the work of Hulme, Mott, Oppenheimer and Taylor (18).

III
Problem Of The Continuous Spectrum

While the line spectrum is relatively easy to photograph and measure, the disintegration electrons of the primary process which went unrecognized for some time constitute the greater portion of the total emission. In the case of Radium E which has no line spectrum, all the emission is of the primary type. Although Schmidt (10) recognized its inhomogenity in 1907, the presence of an abundance of disintegration electrons in all beta emitting bodies was not recognized until 1914 after the experiments of Chadwick (19), who used both an ionization chamber and a "point counter" in place of the photographic plate, as the detector for his spectrometer.

His results plotted as a distribution curve of number vs. momentum, showed on Rutherford's interpretation that there exists a background of these disintegration electrons which is continuous, and rises from the lowest measurable momentum or energy to a maximum and then decreases until a rather definite energy maximum is reached, beyond which there is no further emission. On this continuous background is superimposed the line spectrum of the internal conversion electrons, in all cases except that of Radium E.

In order to determine whether this continuous energy distribution was, so to speak, congenital with respect to the nucleus, or on the contrary introduced after the primary expulsion by some form of encounter between the disintegration product and the outer electrons, Ellis and Wooster (20) measured with a micro-calorimeter the total energy of the beta rays constituting the continuous spectrum of Radium E. This could be accomplished because (as far as was known at the time of the experiment) practically all the energy of emission was confined to these beta rays, the amount from a very feeble

gamma ray being practically negligible. By making the walls of the calorimeter so thick as to absorb the highest energy beta rays it was presumed that all their kinetic energy could be transformed into heat. The results of their experiment indicated that the total energy of disintegration thus measured was 2.6 times SMALLER than the MAXIMUM energy which these electrons were found at times to possess as a result of disintegration; and agreed to within experimental error with the AVERAGE value determined from the distribution curve.

This seemed to be proof conclusive that the electrons were initially emitted with varying energies, otherwise the total energy should have come out closer to the maximum value than to the mean. The validity of the method and values of Ellis and Wooster was later confirmed by Meitner and Orthman (21).

Students of the nucleus were thus confronted with the problem of an identically similar group of parent nuclei all having the same determined energy, disintegrating into product nuclei, likewise of a definitely determined energy, and yet the other member of the disintegration i.e. the electron, coming off from the disintegration process with no unique energy value, but with a continuous energy distribution from practically zero to a certain maximum. Certainly if all factors had been accounted for here was a case of the breakdown of the principle of the conservation of energy.

Another series of experiments and their conclusions led to a result equally difficult to reconcile with the conservation principle.

It is known that the disintegration of the "C" bodies in the three disintegration chains is a branching one, and for the Thorium chain may be represented as follows:

$$\begin{tikzcd}[column sep=small] & & & \text{Th C}' \arrow[dr, "\alpha"] & \\ \text{Th B} \arrow[r, "\beta"] & \text{Th C} \arrow[urr, "\beta"] \arrow[drr, "\alpha"'] & & & \text{Th D (Lead)} \\ & & & \text{Th C}'' \arrow[ur, "\beta"'] & \end{tikzcd}$$

Along the C′ branch the energy of the alpha particle is 8.95 million electron volts, while recent measurements on the end point of the continuous spectrum of Th. C and C″ by Henderson (22) have yielded a value, for the upper limit of the Th. C continuous beta spectrum, of 2.25 M E V.

In the other (C″) branch the fastest alpha ray from Th. C has an energy value of 6.2 M E V, while the upper limit of the C″ spectrum is, from Henderson's measurements, known to be 1.79 M E V. There are in addition two gamma rays emmitted at every disintegration, of energies 2.62 M E V and 0.582 M E V, which must be taken into account.

If one takes the sum of the energies around the two branches one finds:

Th. C Alpha	6.20MEV		Th. C beta max.	2.25MEV
Th. C'' beta Max.	1.79MEV		Th. C' alpha	8.95MEV
" " gamma	2.62MEV			
" " "	0.58MEV			11.20MEV
	11.19MEV			

Thus we see that within limits of experimental error there is a balance of energy around the two paths of the branching disintegration, *but only under the condition* that we consider the *maximum* and not the *average* energy of the beta ray continuous spectrum to be the quantity which enters into the formation of the equality.

IV
Theories Of Beta Disintegration

Ellis and Mott (23) recognized this requirement even before the accurate measurements of Henderson had been made, and used it as an argument for the support of their theory of beta disintegration in which they propose that the upper limit of the continuous beta ray spectrum corresponds to the difference in the binding energies of the initial and final nuclei taking part in the disintegration.

On this theory, if this maximum energy is not given to the beta ray the nucleus is left in an excited state and will return to normal with the emission of a gamma ray. The spectrum of Radium E, which as has been remarked, emits practically no gamma radiation was presumed always to leave its product nucleus, Polonium, in the ground state. And for this reason the Radium E distribution curve was taken to represent the fundamental type, or "single mode of disintegration." Other disintegrations involving considerable gamma radiation were considered to be combinations of two or more of the single modes of disintegration, and from a study of the energy levels of the excited states, as revealed by the gamma rays, the distribution curves for the more complex spectra were thus built up by a process of superposition.

While these composite curves agreed fairly well with the known experimental distribution curves the whole theory completely ignored the problem of lack of energy conservation in the process; in fact the authors baldly state that in the case of Ra E they "do not wish to dwell on what happens to the excess energy in those disintegrations in which the electron is emitted with less than the maximum energy."

In fact they emphasize the point by further pointing out that unless this energy is accounted for there is a breakdown even in the *statistical* conservation of energy. They do however mention in passing that the suggestion of Pauli postulating a third particle of no charge and electronic mass might give the answer to this difficulty, but the matter is carried no further than that.

N. Bohr (24) until recently favored the abandonment of the principle of conservation of energy at least for nuclear processes but in July (1936) relinquished this position in favor of the neutrino hypothesis which we will now consider.

Fermi (25) taking up the suggestion of Pauli, mentioned in connection with the theory of Ellis and Mott, has developed a consistent theory of beta disintegration.

The postulated third particle on which rests the burden of explaining away the energy discrepancy in beta disintegration processes is named the neutrino.

Its main characteristics seem to be negations. It is, if not undetectable, at least undetected by any presently known or applicable means. And its powers of penetration must be such that it can pass through large volumes of matter (such as the walls of the Ellis and Wooster calorimeter) without surrendering a measurable amount of its energy.

It has no charge, and evidence which will be considered later indicates that its mass if not actually zero is negligible compared to that of the electron.

Combining the above hypothesis with those of Heisenberg (26) and Majorana (27) which require only neutrons and protons in the nucleus (with the exclusion of electrons) Fermi was once able to reconcile other difficulties on nuclear statistics, spin and radii as well as the primary one on energy conservation. His theory also leads to a rationalization of the empirical relations discovered by Sargent (28) between the disintegration constant and maximum energy of emission of the continuous beta ray spectrum, whereby all but one of the heavy beta ray emitting naturally radioactive elements are found to lie along two straight lines, if a plot be made of the logarithm of their disintegration constants against the logarithm of the maximum of their continuous beta ray spectra.

The most original assumption adopted by Fermi from Heisenberg is that there is really only *one* heavy particle in nature and that the proton and neutron represent two inner quantum states of this particle. The transition of this heavy particle from the neutron to the proton state is accompanied by the creation of two lighter particles; the electron and the neutrino. The inverse process, that is the conversion of a proton into a neutron may be accomplished either by the absorption of an electron and a neutrino, or the emission of a positron and antineutrino. Thus will both charge and energy be conserved. (It is worth noting that more recent calculations concerning the neutrino indicate that the process involving the absorption of the neutrino is most improbable, the neutrino having to go through 10^{16} kilometers of solid matter (on the average) before capture takes place (29). Thus also does it well fulfill its role as a particle of high penetrating power.)

On the Fermi theory the process involving the generation of disintegration or primary electrons is not to be considered analogous to the emission of alpha particles from the nucleus, but is rather to be likened to the formation of photons which takes place in a radiating body. In fact the equations of Fermi are built up from radiation theory (30). Thus there is no more contradiction in postulating the emission of an electron which did not pre-exist in the nucleus, than there is in stating that a light quantum of a definite $h\nu$ does not exist, as such, in an excited hydrogen atom, (for example) which subsequently emits radiant energy equal to $h\nu$ when the atom passes from a higher to a lower quantum state.

The principal difference between radiation theory and the beta disintegration of Fermi lies in the fact that in the nuclear transition from neutron to proton there must be two particles emitted instead of one photon. This is necessary because the total disintegration energy of the beta decay must be distributed between electron and neutrino, a definite amount always going into the formation of the rest mass of the electron (and neutrino if it have such) and the remainder appearing as the kinetic energy of the two particles, which may be shared between them in continuously varying amounts. This type of distribution gives rise to the continuous beta ray spectrum and the residual energy is carried off by the undetected neutrino. The upper limit for the energy of the beta ray spectrum is therefore $E_{max} = E_0 - (m + \mu)c^2$, where E_0 is the total energy of disintegration, m and μ are the masses of the electron and neutrino and c is the velocity of light.

If the neutrino had appreciable mass then the Fermi theory would require that the energy distribution curve for the electrons should drop suddenly to zero from a definite number of disintegrations at its upper limit, meeting the energy axis with a vertical tangent. This is contradicted by experiment for the curve actually appears to approach the axis almost asympotically. The conclusion is that the mass of the neutrino is certainly smaller than that of the electron and Fermi has taken it, for purposes of calculation, as zero.

Fermi's formula (44) reference (25) gives the disintegration probability for electrons of all possible momenta from elements whose atomic number is not much different from 82.2.

From a detailed consideration of this formulation referred to above, (which will not be repeated here) it must be concluded that the probability of receiving any given amount of energy is the same for both electron and neutrino.

But it has been found from the experimental curves that the electron on the average receives much less than half the maximum energy. Hence the Fermi theory errs in predicting more electrons of higher energy than are actually found.

In addition the theoretical energy distribution curve has a finite intercept on the ordinate or number axis, whereas it has been believed, though on rather slim evidence, that the true curve will pass through the origin.

In other respects the theoretical curve is quite similar to the experimental ones and **Konopinski** and **Uhlenbeck** (31) undertook the modification of the Fermi theory in such a way as to make it conform more closely with experimental data.

This meant introducing a correction which would made the emission of the higher energy neutrinos more probable, and thus result in the displacement of the maximum (number) of the electron distribution curve in the direction of electrons of lower energy. They accomplished this result by the introduction of the derivative of the neutrino wave function with respect to the coordinates, in the place of the wave function itself. The result of this modificaion in the final formula is the appearance of the last factor (which represents the neutrino energy) as a fourth power function rather than a second. And this change is just the one which is necessary to bring the experimental and theoretical maxima into agreement. Since the formula (for nuclei of high atomic number) is otherwise identical, there is still a demand for a finite intercept on the number axis, so that an appreciable number of electrons with zero and near zero energy are predicted.

Kurie, Richardson and Paxton (32) were the first to apply an ingeniously conceived criterion which could be used as a negative norm in judging the validity of the Fermi theory. By plotting the appropriate root of the distribution function against the energy of the emitted electrons any disagreement between the experimental distribution curve and the one predicted theoretically could be determined by the departure of the plot from linearity. Their experimental data for such a plot were taken from photographs of cloud chamber electron tracks. The chamber was placed in a magnetic field and the curvature of the tracks which was inversely proportion to the momentum, could be determined by direct measurement on the photographic plate. Their sources were the artificially radioactivated elements N^{13}, F^{17}, Na^{24}, Si^{31}, P^{32}, Cl, A, and K^{42}. They found that the plot of the Fermi function (with constants adjusted for light elements) did *not* give a straight line, but that by taking the fourth root of the function instead of the second, the resulting graph showed the rectilinear property for the first five of the eight elements examined.

The data from Cl, A, and K^{42} gave points through which two straight lines could be drawn thus giving two intercepts, and indicating according to the authors' interpretation the possibility of a double mode of disintegration. The subsequent work of several experimentalists (33), (34), (35) all seems to support the view that

the Fermi theory fails according to this criterion, whereas in more than the majority of cases examined, including position emitters (B^{9}, B^{11}, C^{13}, N^{15}, O^{17}, B^{8}, O^{11}, N^{11}, C^{12}, Rh, Ag, Dy,) the so-called "K.U." plot resulted in a straight line, which is a confirmation of the correctness of the Konipinski-Uhlenbeck modification of the Fermi theory. The other sources which required two straight lines, indicating a complex type of disintegration, are in some dispute; Gaertner, Turin and Crane (34) finding a single line sufficient for Mn., whereas Brown and Mitchell (35) require two lines to represent the plot of their data on this element. Satisfactory results with I have not as yet been obtained. In addition to Cl, A, and K^{42}; As^{76} requires a double line. There seems to be considerable similarity here between this complex disintegration resulting from the interpretation of the "K.U." plot and the Theory of Ellis and Mott. However of the five elements requiring two lines for the K.U. plot only three (A, As and Mn) are listed as giving gamma rays, a necessary condition for complex disintegration on the Ellis and Mott theory.

While the above type of confirmation has led Bethe and Bacher (29) to accept the Konopinski-Uhlenbeck modification of the Fermi theory as the basis of all their discussions on Beta disintegrations, in their treatise on Nuclear Physics, it cannot be said to have completely solved all its problems. In addition to the difficulty of Nordsieck (36) who maintains that the interaction between neutron and proton in the electron-neutrino field is too small to account for known binding energies as well as scattering effects there have been other criticisms; specifically that the straight line extrapolation to the energy axis gives too high a value to the end point of the energy distribution curve; also that the linearity of the "K.U." plot may be accidental, and is therefore not to be taken as an absolute confirmation of the theory.

Although many investigators had studied beta ray spectra of naturally radioactive bodies, in view of the recent theories considered above and because of the discrepancies existing in the work of previous experimenters, the writer undertook to re-examine the continuous spectrum of Radium E with a magnetic spectrometer of improved design in which sensitive Geiger-Mueller coincidence counters were substituted for photographic detection.

The details of this investigation have been published in the Physical Review (37) to which the reader is referred.

The results may be summarized by saying that the observed energy distribution of electrons yields a non-linear Fermi plot but a linear K.U. one throughout the major portion of the spectrum. This appears to definitely justify the rejection of the original Fermi function, as previously indicated in the case of artificially radioactivated elements, and while the evidence obtained in favor of the Konopinski-Uhlenbeck modification of the Fermi theory is by no means conclusive

it indicates that this theory offers the best explanation, to date, for the mode of disintegration of radioactive bodies which emit electrons or positrons.

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GEOLOGY

AN ALASKAN EXPEDITION.

REV. BERNARD R. HUBBARD, S.J.

Father Bernard Hubbard, S.J., known as "*The Glacier Priest*" has acquired a considerable amount of scientific knowledge of the Arctic Region. His yearly dangerous expeditions are well known; and his friends will be interested to know that at present he is in Alaska, and he intends to remain two years. The purpose of this expedition will be to:

1) Definitely prove the existence of land in that now unknown area north of Siberia and north-west of Point Barrow, Alaska;

2) Record fifty years of Jesuit missionary labour in Alaska with sound and colour motion pictures for use in connection with the fourth centenary of the Society of Jesus in 1940;

3) Live a year with the inhabitants of King Island, whom he calls "the most unspoiled Eskimos in Alaska.";

4) Prove the ability of an all-metal boat of his own design to combat successfully the ice-choked water of Alaska.

A New Fuel

As evidence of the enthusiasm, care and far-sightedness with which Fr. Hubbard goes into his work of exploration, there is the fact that he has proved that seal oil can be used successfully in certain types of diesel engines, and he is prepared to fall back upon that saving knowledge if need comes for it.

Struck with the idea sometime ago, Fr. Hubbard shipped a five-gallon can of seal oil from Alaska to the engineering department of Santa Clara University, where it was proved that it was suitable fuel for a diesel engine. Therefore Father Hubbard's boat is being equipped with a semi-diesel engine, and he will be in a position to benefit by this discovery if his regular fuel should become exhausted.

Explaining why he is certain he will discover land in the yet unexplored Arctic wastes, Father Hubbard says that each year when hundreds of thousands of sea birds gather at Point Barrow to nest, thousands of them do not stop there but continue on northwest and disappear over the horizon. The birds go north to nest in the spring and return south in the autumn.

These birds take the same route north and south each year. They

have not been known to appear at any inhabited place beyond the unexplored wastes, so Fr. Hubbard believes they fly out to land situated in the unexplored region between 200 and 300 miles northwest of Point Barrow.

Moreover, Fr. Hubbard adds, the rim of the Arctic Ocean is known to be lined with islands everywhere except at this point, and this fact would indicate that there should be an island a this place.

As a further argument, Fr. Hubbard adduces the following:

'A few years ago," he tells, "the body of a native was washed ashore at Point Barrow. It was presumably an Eskimo. The body was dressed in a costume which was worn by Eskimos 200 years ago. It could not have been a body preserved that long in the ice, because a body cannot be preserved in moving ice. It would be cut up and torn apart. The body could be identified with no living Eskimos known to-day."

Explaining that it is only a theory, and an attempt to explain the existence of this particular Eskimo, Father Hubbard says it might be that this individual was one of a group of Eskimos living on one of the islands, who, attempting a voyage to Point Barrow in an omiak (a walrus-skin canoe holding about forty people), lost their oars in one of the terrific storms common in that area, and were swept by the northward current into the icy wastes he hopes to explore.

If this is what happened, the Eskimos would have found it impossible to escape from the place to which the current had carried them, and he does not believe they would have survived for any considerable length of time. Therefore, Father Hubbard declares, he is confident that when he finds the land he seeks, he will find that Eskimos have been there before him.

Fr. Hubbard believes that the land which he expects to find will be a rocky island sticking up out of the ice.

Plans for the Journey

Starting out from America at the end of last June, Fr. Hubbard will be two years in the Bering Straits, the Arctic Ocean and the interior of Alaska. He will make his motion picture record of Jesuit missionary work in Alaska in the course of visits to every one of the Jesuit missions in the peninsula. His stay in Alaska will begin with a year on King Island, which, because of the ice, it is absolutely impossible to leave from October to June.

The Rev. Fr. Lafortune, S.J., has converted every one of the 190 inhabitants of the island, and some of these Eskimos will be included in Fr. Hubbard's party when he begins his exploration of the ice fields. Fr. Lafortune is thoroughly familiar with the Eskimo language and has produced a dictionary. He will help Fr. Hubbard to study the language.

Possibly in April, 1939, Fr. Hubbard will set up his readquarters

at Point Barrow and begin his trek into the ice fields. A radio direction beacon will be set up at Point Barrow, its impulses being sent out northwestward—the direction in which the undiscovered land is believed to lie. A short-wave wireless station also will be set up there.

Going into the ice fields, Fr. Hubbard will leave parties of Eskimos about forty miles apart, and it will be their duty to make trips back and forth to keep open a line of communication between one another.

Wireless Directions

After the last party is stationed, Fr. Hubbard and his assistants will make the final dash for the undiscovered land. Members of the party will wear specially-devised helmets which will hold radio phones against the ears. This will enable members of the party to hold to, or get back on, the line pointed out by the radio direction beam.

Members of Father Hubbard's party will include Kenneth Chisholm, former football star at Santa Clara University, who has explored with Father Hubbard for eight years; Ed. Levin, a 240-pound former pugilist, who has been fifty-four times in the ring and has never been knocked out, and is now studying for a degree in philosophy at Stanford University. He is a veteran of seven years' work with Father Hubbard. Bernard Stanley, a younger Santa Clara man, is going as wireless operator.

In the trek from Point Barrow to the undiscovered land, Father Hubbard's party will use sleds and kayaks. They will paddle the kayaks across open water and will haul the sleds themselves over the ice. No dogs will be taken, because it will be a problem to carry enough food for the members of the party alone. An aeroplane is of no use because the ice is too broken.

The boat, which is being made to Fr. Hubbard's specification, is being made of allegheny-metal. It is to be thirty feet long, with a nine-foot beam, and will be propelled by a seventy-five horse-power semi-diesel engine. Its super-structure will be streamlined, and its metal sides will be so highly polished that the ice will not be able to cling to them. In this way, it is expected, the boat will be psuhed up above the ice instead of being crushed if it is caught in any impassable floes.

Father Hubbard is taking 300,000 feet of motion picture film on this expedition.

NEWS ITEMS

MANILA OBSERVATORY, Manila, P.I.

Dr. Bailey Willis, the eminent geologist of Stanford University, spent the months of March, April and May in the Philippines. He made geological reconnaisances, on foot, by airplane and by boat in many parts of the Islands to verify his concepts of the dynamics of the Philippines. He visited the Observatory three times and used its seismic data for a correlation of earthquakes with observed fault lines and planes. He very kindly marked the results of his observations on our set of topographic maps.

Father Deppermann is steadily carrying on his research on typhoons and clouds and has made some important contributions to the subjects. His work is receiving very favorable notice and praise from meteorologists in Europe and America.

Father Deppermann recently published the results of his research in meteorology under the title: "The Weather and Clouds of Manila." In this publication are thirty-seven pages of descriptive matter and one hundred excellent photographs of various types of clouds.

The Bulletin takes this occasion to congratulate the Author on his splendid research in meteorology.

Father Doucette enjoyed the privilege of a trip to Guam and return on the Pan-American Clippers. He left Manila on May 22nd and arrived back on June 3rd. He is expected to favor the *Bulletin* with an account of the flights.

The local press carried a news item recently that Professor Melchor of the Department of Engineering of the University of the Philippines had resigned his position and the resignation had been accepted by the University. A month or so previously the press reported that Professor Melchor is going to M.I.T., Boston, to study Meteorology and Astronomy. These statements may have some significance for the future status of the Observatory.

WESTON COLLEGE

Rev. Michael Joseph Ahern succeeded Harvey J. Skinner of Boston as chairman of the Northeastern Section of the American Chemical Society. Father Ahern was the Catholic speaker at the tercentenary celebration at the Harvard Divinity School.

Seismological Observatory—

Numbers 1 to 5 of the Station Bulletin have been published.

Since July 1st, approximately twenty quakes within a radius of five hundred miles of Weston were recorded. The strongest of these occurred on July 19th on Long Island, July 27th at Rockville, Conn., and September 3th in the vicinity of St. Johns, N.B.

Weston was represented at the 33rd New England Intercollegiate Geological Conference, held in New York, by Fr. Linehan, the only Jesuit and the only priest present.

Members of the Department have given about a dozen lectures in and about Boston.

Father Linehan is taking a course in Structural Geology at Harvard University and Father Smith one in Geophysics at the Massachusetts Institute of Technology. Mr. Langguth is working on an original problem connected with a peculiar long-period disturbance of the seismometer. Mr. Devlin is engaged in seismic observation work.

Physics Department

An advanced Seminar in Theoretical Mechanics is conducted by Fr. T. Smith for the theologians. Attending regularly are Messrs. W. Burns, J. J. Devlin and Langguth.

HOLY CROSS COLLEGE

Chemistry Department—

A program of the Seminars concerning the recent advances in chemistry was recently published. These discussions are presented by the faculty and students of all the departments of chemistry. The meetings are held on successive Mondays from October 25 to May 2. Many interesting papers are announced and they include many branches of advanced chemistry.

LOYOLA COLLEGE, Baltimore, Maryland

Biology Department—

The Mendel Club arranged the seminar program for the present scholastic yeear. These meetings are conducted by the students. Ten papers will be presented from October 29 to May 13. Rev. Joseph S. Didusch is the Faculty Director.

Physics Department—

A new interferometer, Michelson type, was purchased by the department. This will be the first instrument to have a horizontal reading microscope instead of the vertical type. The new idea was suggested by Rev. Thomas H. Quigley, to the manufacturer.

Chemistry Department—

At the National Meeting of the American Chemical Society held at Rochester, N. Y., during the week of September 6, Rev. Richard B. Schmitt read a paper before the Micro-Chemical Section.

The Chemists Club opened its eighth year of series of non-resident lecturers on October 19. Dr. Alexander O. Gettler, Toxicologist of New York City gave an interesting lecture to a large group of students and others on the subject: The Role of Toxicology in Medico-Legal Autopsy. The lecture was presented in Library Hall because of the large attendance.

On November 22, Dr. Ralph Muller, Professor of Physical Chemistry, New York University, delivered a lecture to the members of the Loyola Chemists Club entitled: Electron—Tube Application in Analytical Chemistry. Dr. Muller performed twelve unique experiments which greatly interested the large attendance.

Dr. William M. Thornton, Jr., formerly of The Johns Hopkins University, will continue his research work in our laboratories.

CANISIUS COLLEGE

Biology Department—

The Mendel Club announced its lecture program for 1937-1938. Twelve meetings were arranged for the scholastic year. Some of the prominent speakers are: Dr. Atwell, University of Buffalo Medical School; Dr. Kletzien, State Institute for Malignant Diseases; Dr. Bean, New York State Department of Health and Dr. Simpson, Director of State Institute for the study of Malignant Diseases.

The department has acquired by purchase 325 species of North American moths and butterflies, the nucleus of a proposed representative insect collection. Four Jewell models, the 33, 48 and 72 hrs. pedestal models of the chick embryo, and the cross-section model of the 10 mm. pig have been added to our collection, giving us a complete set of the Jewell pig and chick embryo models. Two reconditioned dissecting binocular microscopes have been added to the equipment of the Genetics course. Our collection of charts has been increased by the purchase of 10 Perrier Cepede charts.

Father Frisch spent eleven weeks of the summer at the Marine Biological Laboratory at Woods Hole, Mass., investigating the experimental adaptation of fresh water protozoa to sea water. The results of this work will be published. Some of the results of his studies of Hunting wasps are being published in the *American Midland Naturalist*. He has supplied Dr. T. M. Sonneborn of Hopkins with three distinct strains of *Paramecium aurelia*, for further tests of the Doctor's theory or sex in Paramecium; he has also supplied Dr. A. Young of Yale with a strain of Blepherisma undulans for comparative cytological study of endomixis in this species.

Dr. A. P. Lorz, the Assistant Professor, spent the entire summer at the laboratory of the Department of Genetics of the Carnegie Institute of Washington at Cold Spring Harbor, Long Island. He cooperated with the staff in cytological investigations of Datura. His work on "Cytological investigations on five chenopodiaceous genera with special emphasis on cromosome morphology and somatic doubling in Spinacia" is being published by *Cytologia*, the international genetics journal (Japan). He has been consulted by the Texas Agricultural Experiment Station on the methods of growing Spinacia for cytological studies and he is cooperating with them.

BOSTON COLLEGE

Biology Department—

The Biology Department this year has the largest enrollment in the history of the college. There are 24 Senior Pre-meds, 20 Junior B.S., 30 A.B. Juniors, and 10 Sophomores B.S.; besides there are 142 taking an Elective Junior Course in Cultural Biology. These numbers mean that the lecture-room or the laboratory is almost continually in use from the first hour on Monday to the last hour on Friday. In addition to this, there is a Saturday morning contingent from the Extension School.

Those who have been familiar with the Museum, formerly on the first floor of the Science Building, will be interested to know that this summer, in order to make space for much needed class-rooms, the Museum was dismantled, and the contents all removed to the Biology corridors, where they provoke much more investigation than ever before.

FORDHAM UNIVERSITY

Chemistry Department—

There are thirty-six Graduate Students registered in Chemistry.

The entire Chemistry building was painted this summer with a special fume-resisting paint.

The complete A. S. T. M. Journals and the Proceedings of the Society of Experimental Biology and Medicine have been procured for the Chemistry library. The J. A. C. S. is now complete to 1897.

Papers published from the Fordham Chemical Laboratories are as follows: six in the September J. A. C. S.; three in the September and one in the Octotber issue of the Journal of Biochemistry. There was also one paper published in the September Microchemie.

Additional laboratory tables and complete digestion and distillation Kjeldahl units have just been acquired.

A course in the chemistry of Nutrition, leading to the M.A. degree is being given under the direction of Dr. Jacob A. Stekol; he

gives the courses in nutrition, biochemistry and food analysis. The students in this course also receive instruction in bacteriology (as applied to food inspection) from Dr. Leonard Piccoli of the Fordham School of Pharmacy. Dr. Piccoli also gives the course in Anatomy and Physiology to the graduate students in biochemistry.

A new course in advanced inorganic chemistry is offered this year by Dr. Walter A. Hynes, including the periodic properties of the elements of interest to the analytical and physical chemist.

Mr. Spikes, president of the N. Y. Microchemical Society appointed Father Power chairman of a committee for the discussion of the Pregl methods. The first meeting of this committee was held October 28th, with Father Power presiding, at the office of Mr. R. L. Van Name, the dealer in microchemical equipment at 95 Madison Avenue.

Biology Department.—

The latest addition to our laboratory is a device to show various phases of cell division "in actu". The plans were made by this department, and the contrivance itself is the work of a skilled mechanic. It shows first four V-bent chromosomes in the equatorial plate, together with the spindle and the centrosomes in aster formation. Then, by starting a mechanism, the chromosomes are divided lengthwise, and finally one-half of the now doubled number of chromosomes is moved up to one pole (marked by one of the centrosomes) of the cell; the second half to the other pole. The structure is expected to be of help to the students with a poor imaginative faculty who find it difficult—if not altogether impossible—to grasp the actual process of cell division. Soon we hope to publish in this *Bulletin* some pictures with an exact description of our "*mitosis apparatus*".

NOTICE

ANNUAL MEETING

OF THE

NATIONAL COUNCIL

OF

JESUIT SCIENTISTS

WEDNESDAY, DECEMBER 29, 1937

CATHEDRAL SCHOOL - 14TH AND MERIDIAN STS.

INDIANAPOLIS, INDIANA

AMERICAN ASSOCIATION FOR THE
ADVANCEMENT OF SCIENCE
CONVENTION

DECEMBER 27 TO 31, 1937